Thermal Radiation Effects on a Normal Shock Wave Incident to a Plane Wall

Michael Lynn Smith*
United States Air Force Academy, Colo.

Nomenclature

A_i = (i = 6, 7, 8, 9, 25, 26) constants which are functions of strength of step shock; values defined in Ref. 3
Bo = Boltzmann number
M_s = isentropic Mach number seen by shock-fixed observer
$\bar{p}$ = dimensionless pressure normalized by $\rho_{oa} W_{So}^2$
$\bar{Q}^R$ = dimensionless total radiation heat flux normalized by $\rho_{oa} W_{So}^2$
$\bar{q}^R$ = dimensionless radiation heat flux because of wall perturbations, normalized by $\rho_{oa} W_{So}^3$
R = ideal gas constant
$\bar{T}$ = dimensionless temperature normalized by T_{ob}
$\bar{t}_a$ = dimensionless time, normalized by $1/(\alpha_{oa} W_{So})$
$\bar{u}$ = dimensionless gas velocity normalized by W_{So}
W_{So} = unperturbed propagation speed of incident shock
$\bar{W}_S'$ = dimensionless perturbation in propagation speed of incident shock, normalized by W_{So}
x = distance measured from shock
x_0 = initial distance from incident shock to wall, $x_0 \rightarrow -\infty$
y_a = distance measured from wall
y_0 = initial distance from wall to incident shock, $y_0 \rightarrow \infty$
α = volumetric absorption coefficient of gas
γ = specific heat ratio
ϵ_w = frequency independent hemispherical emittance of wall
η = dimensionless distance measured from shock, $\eta \equiv \alpha_0 x$
η_{wo} = dimensionless distance from incident shock to wall, $\eta_{wo} \leq \eta_o$
ψ = ratio of perturbation parameters, $\psi = (1/Bo_b)/(1/Bo_d)$
$\bar{\rho}$ = dimensionless mass density of gas, normalized by ρ_{ob}
ρ_w = frequency independent hemispherical reflectance of wall
σ = Stefan-Boltzmann constant

Subscripts

a = conditions in region between wall and incident shock
b = conditions in region between incident shock and infinity
c = conditions in region between reflected shock and infinity
d = conditions in region between wall and reflected shock
o = undisturbed conditions
w = wall property
1 = first-order perturbed variable

Superscripts

$'$ = perturbed variable
$-$ = dimensionless variable
$\hat{}$ = dummy integration variable

Theme

THE results of a theoretical study of the effects of low-intensity thermal radiation on the propagation of a normal shock wave incident to a plane, grey, radiating wall are given. The shock is treated as a discontinuity embedded in a structure due to radiation, and the effects of self-absorption and emission in the gas on both sides of the shock are included. The initial or free shock[1] structure is caused by radiation from the hot gas behind the shock. This structure is modified as the shock approaches the wall because of emission, absorption, and reflection by the wall.

All nonequilibrium phenomena (conduction, chemistry, ionization, viscous effects, etc.), with the exception of radiation, are neglected; and, in addition to being grey, the gas is assumed to be in quasi-thermodynamic equilibrium and thermally and calorically perfect. The differential approximation[1] is used in solving for the radiation heat flux, and scattering, radiation energy density, and radiation pressure are neglected.

A first-order method of characteristics solution about the initial profile is obtained analytically for the velocity, temperature, pressure, density, radiation heat flux, and shock propagation speed by assuming that all variables can be expressed in power series expansions of the reciprocal of the Boltzmann number. It is shown that the flowfield perturbations for the incident shock are of the same order as those for the reflected shock.

Contents

Olfe[2] treats the radiating shock problem for the more general case of a nongrey wall and gas. However, the focus of his attention is the region behind the reflected shock, and only implicit allowance is made for self-absorption and emission on either side of the incident shock. Here, explicit results are derived for these regions. The coupling effects of incident shock structure and the effects of the wall on the initial structure of the reflected shock are shown.

Consider the configuration shown in Fig. 1. An infinite plane wall is located at $y_a = 0$, and a semi-infinite expanse of gas occupies the region $y_a > 0$. The wall and adjacent gas are initially at the same temperature $T_{oa} = T_w$. For convenience, assume a grey wall such that the total hemispherical emittance and reflectance are related by $\epsilon_w = 1 - \rho_w$. At a large distance ($y_a = y_o >> 0$), we generate a normal shock wave with direction of travel toward the wall. As the shock approaches the wall, variations in the free-shock profile will appear. The net profile is determined by the sum of the free-shock and wall-induced contributions.

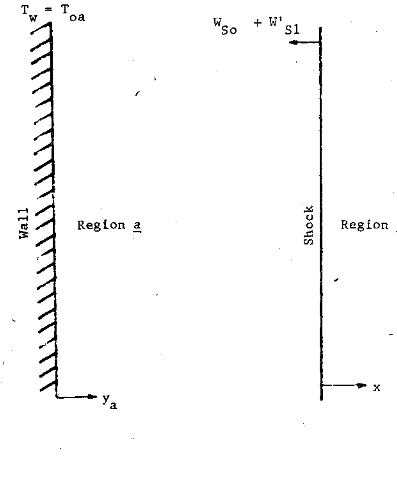

Fig. 1 Incident shock configuration.

Received June 11, 1976; synoptic received Jan. 3, 1977; revision received May 16, 1977. Full paper available from National Technical Information Service, Springfield, Va., 22151 as N77-23409 at the standard price (available upon request).

Index category: Radiatively Coupled Flows and Heat Transfer.

*Captain, U.S. Air Force; Associate Professor, Department of Aeronautics. Member AIAA.

To solve for the wall-induced profile, we divide the solution domain into two regions, *a* and *b*, and use the method of characteristics technique to solve for the flowfield in each region. The condition of zero velocity at the wall and Rankine-Hugoniot conditions across the discontinuous portion of the shock serve as the boundary conditions in regions *a* and *b*, respectively.

For region *a*, the continuity, momentum, energy, and state equations for the first order (in the reciprocal of the Boltzmann number, $Bo_b \equiv \gamma R \rho_{ob} u_{ob} / (\gamma - 1)\sigma T_{ob}^3$) nondimensional wall-induced perturbations to the flow properties in a wall-fixed coordinate system are[3]

$$\frac{\partial \bar{\rho}'_{al}}{\partial \bar{t}_a} - \frac{\rho_{oa}}{\rho_{ob}} M_{Soa}^2 \frac{\partial \bar{p}'_{al}}{\partial \bar{t}_a} = (\gamma - 1) M_{Soa}^2 \frac{\rho_{oa}}{\rho_{ob}} \frac{\partial \bar{q}'^R_{al}}{\partial \bar{y}_a} \tag{1a}$$

$$\frac{\partial \bar{u}'_{al}}{\partial \bar{t}_a} + \frac{\partial \bar{p}'_{al}}{\partial \bar{y}_a} = 0 \tag{1b}$$

$$\frac{\partial \bar{p}'_{al}}{\partial \bar{t}_a} + \frac{1}{M_{Soa}^2} \frac{\partial \bar{u}'_{al}}{\partial \bar{y}_a} = -(\gamma - 1) \frac{\partial \bar{q}'^R_{al}}{\partial \bar{y}_a} \tag{1c}$$

$$\bar{T}'_{al} = \frac{W_{So}^2}{RT_{ob}} \bar{p}'_{al} - \frac{\rho_{ob}}{\rho_{oa}} \frac{T_{oa}}{T_{ob}} \bar{\rho}'_{al} \tag{1d}$$

Using the differential approximation,[1] the first-order equations[3] for the total radiation heat flux in a shock-fixed coordinate system are

$$\frac{\partial^2 \bar{Q}'^R_{al}}{\partial \eta_a^2} - 3\bar{Q}'^R_{al} = 0, \quad \frac{\partial^2 \bar{Q}'^R_{bl}}{\partial \eta_b^2} - 3\bar{Q}'^R_{bl} = 0 \tag{2a,b}$$

where $\eta_a \equiv \alpha_{oa} x$, $\eta_b \equiv \alpha_{ob} x$, and x is related to y_a by

$$y_a = x - x_0 - \int_0^t (W_{So} + W'_S) \, d\hat{t}$$

The boundary conditions for Eq. (2) are determined by noting that the heat flux entering the gas from the wall is the sum of emitted and reflected radiation, the shock discontinuity is transparent, and the radiation heat flux remains finite infinitely far behind the shock.

The characteristic equation determined by the second and third relations in Eq. (1) is

$$d\bar{p}'_{al} \pm \frac{1}{M_{Soa}} d\bar{u}'_{al} = -(\gamma - 1) \frac{\partial \bar{q}'^R_{al}}{\partial \bar{y}_a} d\bar{t}_a \tag{3}$$

The equations of the characteristics are $(d\bar{t}_a / d\bar{y}_a) = \pm M_{Soa}$.

Substituting the wall-perturbed portion of the solution to the first relation of Eq. (2) into Eq. (3) and integrating yields

$$\bar{P}'_{al} = A_6 \exp[\sqrt{3}(2\eta_{wo} - \eta_a)] + A_7 \exp\{\sqrt{3}[(M_{Soa} + 1)\eta_{wo} - M_{Soa}\eta_a]\} \tag{4a}$$

$$\bar{u}'_{al} = A_6 \exp[\sqrt{3}(2\eta_{wo} - \eta_a)] + M_{Soa} A_7 \exp\{\sqrt{3}[(M_{Soa} + 1)\eta_{wo} - M_{Soa}\eta_a]\} \tag{4b}$$

where we have expressed the results in the shock-fixed $(\eta_a, \bar{t}_a)$ coordinate system and have used the boundary condition that the velocity is zero at the wall. Note that, in accomplishing the coordinate system conversion from the $(\bar{y}_a, \bar{t}_a)$ to the $(\eta_a, \bar{t}_a)$ system, we have neglected the contribution of the nondimensional shock speed perturbation $(\bar{W}'_S)$ to the result, since this will be a second-order effect if we assume (as for all other variables) that

$$\bar{W}_S = 1 + (1/Bo_b)\bar{W}'_{S_l} + 0(1/Bo_b^2)$$

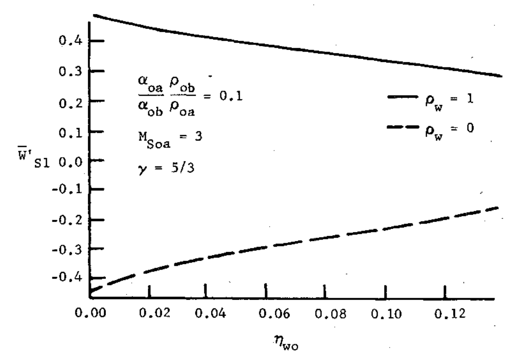

Fig. 2 Incident shock speed perturbation.

The wall-induced density and temperature perturbations determined from the first and last relations of Eq. (1) are

$$\bar{\rho}'_{al} = \frac{\rho_{oa}}{\rho_{ob}} A_6 \exp[\sqrt{3}(2\eta_{wo} - \eta_a)] + \frac{\rho_{oa}}{\rho_{ob}} M_{Soa}^2 A_7 \exp \{\sqrt{3}[M_{Soa} + 1)\eta_{wo} - M_{Soa}\eta_a]\} \tag{5a}$$

$$\bar{T}'_{al} = A_8 \exp[\sqrt{3}(2\eta_{wo} - \eta_a)] + A_9 \exp\{\sqrt{3}[(M_{Soa} + 1)\eta_{wo} - M_{Soa}\eta_a]\} \tag{5b}$$

Equations (4) and (5) constitute the solutions for the first-order wall-induced flowfield perturbations in the region ahead of the incident shock. The total first-order perturbations are given by combining these expressions with the free-shock perturbations.[3]

A parallel development[3] leads to similar expressions for the pressure, velocity, density, and temperature in region *b*. Application of the Rankine-Hugoniot conditions across the shock discontinuity leads to the following expression for the first-order shock speed perturbation:

$$\bar{W}'_{Sl}(\bar{t}_a) = A_{25} \exp\{2\sqrt{3}\eta_{wo}\} + A_{26} \exp\{\sqrt{3}(M_{Soa} + 1)\eta_{wo}\} \tag{6}$$

Figure 2 shows the first-order shock speed perturbation for a black and a perfectly reflecting wall. We note that for the perfectly reflecting wall, the perturbation shock speed increases from a zero value (free-shock condition) to a maximum at the wall. The behavior is just the opposite for the black wall, since the black wall acts as a heat sink, absorbing radiation which would tend to strengthen the shock.

Conditions present when the incident shock is at the wall will serve as initial conditions for the reflected shock problem. In order to get a small perturbation solution, the temperature of the wall must jump to T_{od}, the classical step shock temperature downstream of the reflected shock.

The appropriate expansion parameter is $1/Bo_d = \gamma R \rho_{od} u_{od} / / \gamma - 1)\sigma T_{od}^3$. It can be shown[3] that $1/Bo_b = \psi(1/Bo_d)$, where $\psi \equiv (T_{oc}/T_{od})^{5/2}(\rho_{od}/\rho_{oc}) M_{Sod} M_{Soc}$. Thus, the flowfield perturbations as well as the perturbation to the shock propagation speed for the reflected shock will be of the same order as their counterparts for the incident shock.

References

[1] Vincenti, W. G. and Kruger, C. H., Jr., *Introduction to Physical Gas Dynamics*, Wiley, New York, 1965, pp. 436-520.

[2] Olfe, D. B., "The Influence of Radiant-Energy Transfer on One-Dimensional Shock Wave Propagation," *Supersonic Flow, Chemical Processes and Radiative Transfer*, Macmillan, New York, 1964, pp. 355-373.

[3] Smith, M. L., "Effects of Thermal Radiation on the Motion of a Normal Shock Wave During Reflection from a Radiating Wall," Ph.D. dissertation, May 1973, University of Texas, Austin, Texas.